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Dyonic Masses from Conformal Field Strengths in D even Dimensions.

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Abstract

We show that D/2–form gauge fields in D even dimensions can get a mass with both electric and magnetic contributions when coupled to conformal field–strengths whose gauge potentials are $\frac{D-2}{2}$– forms. Denoting by e^I_Λ and $m^{I\Lambda}$ the electric and magnetic couplings, gauge invariance requires: $e^I_\Lambda m^{J\Lambda} \mp e^J_\Lambda m^{I\Lambda} = 0$, where $I, \Lambda = 1 \cdots m$ denote the species of gauge potentials of degree $D/2$ and gauge fields of degree $D/2-1$, respectively. The minus and plus signs refer to the two different cases $D = 4n$ and $D = 4n+2$ respectively and the given constraints are respectively $\mathrm{Sp}(2m)$ and $\mathrm{O}(m, m)$ invariant. For the simplest examples, ($I, \Lambda = 1$ for $D = 4n$ and $I, \Lambda = 1, 2$ for $D = 4n+2$) both the e, m quantum numbers contribute to the mass $\mu = \sqrt{e^2 + m^2}$. This phenomenon generalizes to D even dimensions the coupling of massive antisymmetric tensors which appear in $D = 4$ supergravity Lagrangians, which derive from flux compactifications in higher dimensions. For $D = 4$ we give the supersymmetric generalization of such couplings using $N = 1$ superspace.

1 Introduction

It is well known that in $D = 4$ dimensions a massless antisymmetric tensor $B_{\mu\nu}$ is "dual" to a massless scalar, while in the massive case [1] it is dual to a massive vector. In this note we consider a generalization of this phenomenon in D even dimensions when a $D/2$–form receives contribution to the mass both from an "electric" and a "magnetic" type of coupling. Denoting by e^I_Λ and $m^{I\Lambda}$ the "electric" charges and the "magnetic" charges respectively, such couplings are of the type $e^I_\Lambda F^\Lambda \wedge B_I$, where $F^\Lambda = dA^\Lambda (I, \Lambda = 1 \cdots m)$ are conformal field–strengths [2] and $\frac{1}{2} m^{I\Lambda *} F_\Lambda \wedge B_I$ [3]. Here and in the following we use the differential form language.

These two couplings have in fact different origin, the former being the four dimensional version of the Green–Schwarz (geometrical) coupling [4] being gauge invariant under $\delta A^\Lambda = d\phi^\Lambda$ and $\delta B_I = d\Lambda_I$. The latter comes from certain flux or Scherk–Schwarz compactifications, and it can be made gauge invariant (when $e^I_\Lambda = 0$) only if $\delta A^\Lambda = -m^{I\Lambda}\Lambda_I$ and a $B_{I\,\mu\nu}$ mass term $\frac{1}{2} m^{I\Lambda} m^{J\Lambda} B_I \wedge^* B_J$ is added [5].
However it has recently been noted in compactification of Type IIB with one tensor field [5] and, more generally, in the context of $N = 2$, $D = 4$ supergravity coupled to vector multiplets and (an arbitrary number of) tensor multiplets [6], that one can have both "electric" and "magnetic" type of mass terms, namely the linearized Lagrangian has the following form[1] :

$$\begin{aligned} \mathcal{L}^{(D=4)} &= -\frac{1}{2} H_I \wedge^* H_I + \frac{1}{2} m^{\Lambda I} m^{\Lambda J} \left(B_I + (m^{-1})_{\Gamma I} F^\Gamma \right) \wedge^* \left(B_J + (m^{-1})_{\Delta J} F^\Delta \right) \\ &\quad -\frac{1}{2} e^I_\Lambda m^{\Lambda J} B_I \wedge B_J - e^I_\Lambda B_I \wedge F^\Lambda. \end{aligned} \tag{1.1}$$

It is immediate to verify that this Lagrangian is invariant under the gauge transformations

$$\delta A^\Lambda = -m^{I\Lambda} \Lambda_I, \quad , \quad \delta B_I = d\Lambda_I \tag{1.2}$$

provided the condition

$$e^I_\Lambda m^{J\Lambda} - e^J_\Lambda m^{I\Lambda} = 0 \tag{1.3}$$

is satisfied [5, 6] (this condition is void in reference [5] since in that case $I = 1$). Note that this condition is Sp$(2m)$ invariant.

[1]We used the fact that for a k–form $\omega^{(k)}$ we have $\omega^{(k)} \wedge {}^*\omega^{(k)} = (-1)^{D-t}\, k!\, \omega_{\mu_1 \ldots \mu_k} \omega^{\mu_1 \cdots \mu_k} \sqrt{|g|}\, dx^1 \wedge \ldots \wedge dx^D$, where t is the space-time signature of the metric chosen to be "mostly minus".

In the following we discuss the generalization of this $D = 4$ case to any even D space–time dimensions.

We first observe that for $D = 4n$, $n > 1$, the above properties of the theory remain the same as in the $n = 1$ case. This appears to be evident if one uses the differential forms language. On the other hand, for $D = 4n + 2$, the combined presence of electric and magnetic mass terms, requires now the condition:

$$e^I_\Lambda m^{J\Lambda} + e^J_\Lambda m^{I\Lambda} = 0. \tag{1.4}$$

This is so because the term $-\frac{1}{2}e^I_\Lambda m^{J\Lambda} B^I \wedge B_J$ whose variation must cancel the variation of the term $-e^I_\Lambda B_I \wedge F^\Lambda$ under the combined gauge transformation:

$$\delta B_I = d\Lambda_I, \quad , \quad \delta A^\Lambda = -m^{i\Lambda}\Lambda_I \tag{1.5}$$

is symmetric for $D = 4n$ and antisymmetric for $D = 4n + 2$. This explains why for $D = 4n$ we can have $I, \Lambda = 1$, but for $D = 4n+2$ the simplest case is $I, \Lambda = 1, 2$ with $e^I_\Lambda = e\epsilon^I_\Lambda$, where $\epsilon = -(\epsilon)^T, \epsilon^2 = -1$, and $m^{I\Lambda} = m\delta^{I\Lambda}$. Only when $m^{I\Lambda} = 0$ we have no restriction on the e^I_Λ and we can also $I, \Lambda = 1$ also for $D = 4n + 2$. Note that equation (1.4 is $\mathrm{O}(m, m)$ invariant. The difference in the invariance groups in the equations (1.3) and (1.4)is related to the duality rotations of conformal field–strengths of degree $D/2$ for rhe $d = 4n$ and $D = 4n + 2$. Gauge invariant mass terms also exist in odd dimensions as is discussed in [8].

2 Massive gauge fields in even dimensions

Let us extend the Lagrangian (1.1) of the four dimensional case. If we take $D = 4n$ the Lagrangian has exactly the same form where now $F^\Lambda = dA^\Lambda$ and B_I are $2n$–forms and $H_I = dB_I$ are $2n + 1$–forms. In the simplest case discussed above, namely if we take just one F field–strength and one B gauge field, the Lagrangian takes the form:

$$\begin{aligned}\mathcal{L} = -\frac{1}{2}H \wedge^* H + \frac{1}{2}m^2\left(B + (m^{-1})F\right) \wedge^* \left(B + (m^{-1})F\right) \\ -\frac{1}{2}em\left(B + (m^{-1})F\right) \wedge \left(B + (m^{-1})F\right).\end{aligned} \tag{2.6}$$

where we have added the total derivative $-\frac{e}{2m}F \wedge F$. In this form the Lagrangian is manifestly invariant under the gauge transformations (1.2) since

the quantity $B + m^{-1}F$ is gauge invariant. By redefining

$$B + m^{-1}dA \longrightarrow B \tag{2.7}$$

the Lagrangian takes the simplest form:

$$\mathcal{L}^{D=4n} = -\frac{1}{2}H \wedge^* H + \frac{1}{2}m^2 B \wedge^* B - \frac{1}{2}emB \wedge B. \tag{2.8}$$

For $D = 4n{+}2$ we also take the simplest case, namely we consider two F's and two B's setting $I, \Lambda = 1, 2$ with $e^I_\Lambda = e\epsilon^I_\Lambda$ and $m^{I\Lambda} = m\delta^{I\Lambda}$. For notational simplicity we do not write the indices $I, \Lambda = 1, 2$ explicitly, treating F and B as two–dimensional vectors and ϵ^I_Λ as the 2×2 antisymmetric matrix $\epsilon, \epsilon^2 = -1$. Adding as before the total derivative $-\frac{e}{2m}F^T \wedge \epsilon F$, the Lagrangian for $D = 4n + 2$, after the redefinition (2.7) has been performed, takes the following form:.

$$-\mathcal{L}^{D=4n+2} = -\frac{1}{2}H^T \wedge^* H + \frac{1}{2}m^2 B^T \wedge^* B - \frac{1}{2}emB^T \wedge \epsilon B. \tag{2.9}$$

where the overall minus sign with respect to the $D = 4n$ case is due to the requirement of positive kinetic energy and mass squared.
From the Lagrangians (2.8),(2.9) we obtain the following equations of motion respectively:

$$d^*dB + m^{2*}B - emB = 0 \quad , \quad D = 4n \tag{2.10}$$

$$d^*dB - m^{2*}B + em\epsilon B = 0 \quad , \quad D = 4n + 2 \tag{2.11}$$

The integrability conditions of these equations can be written as the transversality conditions

$$d^*\left(B + \frac{e}{m}{}^*B\right) = 0 \quad , \quad D = 4n \tag{2.12}$$

$$d^*\left(B - \frac{e}{m}\epsilon\,{}^*B\right) = 0 \quad , \quad D = 4n + 2 \tag{2.13}$$

Using the constraints (2.12), (2.13) we can write the equations (2.10),(2.11) as equations for ${}^*B$ and, taking linear combinations, we obtain:

$$d^*d\left(B + \frac{e}{m}{}^*B\right) + (e^2 + m^2)^*\left(B + \frac{e}{m}{}^*B\right) = 0 \quad , \quad D = 4n \tag{2.14}$$

$$d^*d\left(B - \frac{e}{m}{}^*\epsilon B\right) + (e^2 + m^2)^*\left(B - \frac{e}{m}{}^*\epsilon B\right) = 0 \quad , \quad D = 4n + 2 \tag{2.15}$$

In deriving these equations we have used the property that, in even dimensions, the Hodge star operator on a p–form $\omega^{(p)}$:

$$
{}^*\omega^{(p)} = \frac{1}{(D-p)!}\omega_{\mu_1\cdots\mu_p}\sqrt{-g}\epsilon^{\mu_1\cdots\mu_p}{}_{\nu_1\cdots\nu_{D-p}}dx_1^{\nu}\wedge\cdots\wedge dx^{\nu_{D-p}} \tag{2.16}
$$

satisfies the relation

$$
{}^{**}\omega^{(p)} = (-1)^{p+1}\omega^{(p)} \tag{2.17}
$$

Taking the Hodge– star of equations (2.14), (2.15) and recalling that the Klein–Gordon operator $\Box$ is defined as

$$
\Box = \delta d + d\delta \quad , \quad \delta = -{}^*d^* \tag{2.18}
$$

we obtain the equations of motion in their standard form:

$$
\Box\left(B + \frac{e}{m}{}^*B\right) + (e^2+m^2)^*\left(B + \frac{e}{m}{}^*B\right) = 0 \quad , \quad D = 4n \tag{2.19}
$$

$$
\Box\left(B - \frac{e}{m}{}^*\epsilon B\right) + (e^2+m^2)^*\left(B - \frac{e}{m}{}^*\epsilon B\right) = 0 \quad , \quad D = 4n+2 \tag{2.20}
$$

The equations (2.19),(2.20), together with the transversality conditions (2.12),(2.13), describe a massive $D/2$–form of mass $\mu = \sqrt{e^2+m^2}$. Note that for $D = 4n+2$ the field B is a 2–vector $B = (B_1, B_2)^T$, each component being a $(2n+1)$–form.

3 The dual formulation

In the massless case it is known, by Poincarè duality, that a massless $D/2$–form is "dual" to a massless $D/2-2$–form. For example in $D = 4$ a 2–form is dual to a scalar and in $D = 6$ a 3–form is dual to a vector.

Here we show that a massive $D/2$–form, described in the previous section, is dual to a massive $D/2-1$–form. As previously discussed a doubling of the $D/2$ gauge potential is required for $D = 4n+2$ when both electric and magnetic masses are present in the theory.

The process of dualization at the level of the Lagrangian requires the first–order formalism for the gauge potential B and it is a straightforward generalization of the method used in references [7, 5]. Let us discuss separately the two cases $D = 4n$ and $D = 4n+2$.

3.1 $D = 4n$

The Lagrangian (2.6) can be dualized by rewriting it in first order form with B and H independent $(2n)$– and $(2n+1)$–forms, and A and F independent $(2n{-}1)$– and $(2n)$–forms, respectively [7], and enforcing the relations $H = dB$ and $F = dA$ by suitable Lagrangian multipliers ρ and ξ which are $(2n)$– and $(2n-1)$–forms respectively. We start with:

$$\begin{aligned}\mathcal{L}^{(D=4n)} &= -\frac{1}{2}H \wedge^* H + \frac{1}{2}m^2(B + \frac{1}{m}F) \wedge^* (B + \frac{1}{m}F) \\ &\quad -\frac{1}{2}em(B + \frac{1}{m}F) \wedge (B + \frac{1}{m}F) + \rho \wedge \left(H - d(B + \frac{1}{m}F)\right) \\ &\quad +\xi \wedge (F - dA) \end{aligned} \tag{3.21}$$

The original Lagrangian (2.6) is retrieved by imposing the equations of motion of ρ and ξ. The dual Lagrangian $\mathcal{L}^{D=4n}_{Dual}$ is instead obtained by varying the forms H and B. One obtains:

$$\frac{\delta \mathcal{L}}{\delta H} = 0 \longrightarrow {}^*H = -\rho \rightarrow H = -{}^*\rho \tag{3.22}$$

$$\frac{\delta \mathcal{L}}{\delta B} = 0 \longrightarrow m^{2*}(B + \frac{1}{m}F) - em(B + \frac{1}{m}F) = d\rho \tag{3.23}$$

$$\frac{\delta \mathcal{L}}{\delta F} = \xi + m^*(B + \frac{1}{m}F) - e(B + \frac{1}{m}F) = \frac{1}{m}d\rho \tag{3.24}$$

$$\frac{\delta \mathcal{L}}{\delta A} = 0 \longrightarrow d\xi = 0 \tag{3.25}$$

From the previous equations we easily find

$$\xi = 0 \tag{3.26}$$

$$B + \frac{1}{m}F = -\frac{1}{e^2 + m^2}\left(\frac{e}{m}d\rho +^* d\rho\right) \tag{3.27}$$

$${}^*(B + \frac{1}{m}F) = -\frac{1}{e^2 + m^2}\left(\frac{e}{m}{}^*d\rho - d\rho\right) \tag{3.28}$$

We redefine $(B + \frac{1}{m}F) \longrightarrow B$. Then Hodge–starring (3.23) and combining with (3.23), we obtain:

$$B = -\frac{1}{e^2 + m^2}\left(\frac{e}{m}d\rho +^* d\rho\right) \tag{3.29}$$

$${}^*B = -\frac{1}{e^2 + m^2}\left(\frac{e}{m}{}^*d\rho - d\rho\right) \tag{3.30}$$

Substituting in (3.21) one finds:

$$\mathcal{L}^{D=4n}_{(Dual)} = \frac{1}{2}\left(\frac{1}{e^2+m^2}\, d\rho\wedge^* d\rho +^* \rho\wedge\rho\right) \tag{3.31}$$

which is indeed the Lagrangian for a massive $2n$–form $\overline{\rho} = \rho(e^2+m^2)^{-1/2}$. The equations of motion are:

$$d^* d\rho - (e^2+m^2)^*\rho = o \tag{3.32}$$

together with the transversality condition $d^*\rho = 0$. Using the definition of the Klein–Gordon operator (2.18) we obtain in the standard formalism for the divergenceless field ρ:

$$\Box\rho + (e^2+m^2)\rho = 0 \tag{3.33}$$

3.2 D=4n+2

The Lagrangian in first order formalism is now:

$$\begin{aligned}\mathcal{L}^{D=4n+2} \;=\; & \frac{1}{2}H^T\wedge^* H - \frac{1}{2}m^2(B+\frac{1}{m}F)^T\wedge^*(B+\frac{1}{m}F) \\ & +\frac{1}{2}e\, m(B+\frac{1}{m}F)^T\wedge\epsilon(B+\frac{1}{m}F) - \rho^T\wedge\left(H - d\,(B+\frac{1}{m}F)\right) \\ & +\xi^T\wedge(F-dA)\end{aligned} \tag{3.34}$$

The original Lagrangian (2.9)is retrieved as before from the equation of motion of ρ.
To obtain the dual Lagrangian we proceed as in the former case $D=4n$ by varying H, B, F and A. We obtain:

$$\frac{\delta\mathcal{L}}{\delta H} = 0 \longrightarrow {}^*H = \rho \rightarrow H = -{}^*\rho \tag{3.35}$$

$$\frac{\delta\mathcal{L}}{\delta B} = 0 \longrightarrow m^{2*}(B+\frac{1}{m}F) - e\,m\,\epsilon(B+\frac{1}{m}F) = d\rho \tag{3.36}$$

$$\frac{\delta\mathcal{L}}{\delta F} = 0 \longrightarrow m^*(B+\frac{1}{m}F) - e\,\epsilon(B+\frac{1}{m}F) + \xi = \frac{1}{m}d\rho \tag{3.37}$$

$$\frac{\delta\mathcal{L}}{\delta A} = 0 \longrightarrow d\xi = 0 \tag{3.38}$$

Redefining $B + m^{-1}F \longrightarrow B$ and proceeding as before we find the expressions of B and ${}^*B$:

$$\begin{aligned} \xi &= 0 && (3.39) \\ B &= \frac{1}{e^2+m^2}\left(\frac{e}{m}\epsilon d\rho +^* d\rho\right) && (3.40) \\ {}^*B &= -\frac{1}{e^2+m^2}\left(\frac{e}{m}\epsilon^* d\rho + d\rho\right) && (3.41) \end{aligned}$$

Substituting in equation (3.34)we find:

$$\mathcal{L}^{D=4n+2}_{(Dual)} = -\frac{1}{2}\left(\frac{1}{e^2+m^2}\, d\rho^T \wedge^* d\rho -^* \rho^T \wedge \rho\right) \tag{3.42}$$

whose equations of motion are:

$$d^* d\rho + (e^2+m^2)^* \rho = o \tag{3.43}$$

together with the transversality condition $d^*\rho = 0$. In the usual formalism the equation (3.43) for the divergeless field ρ becomes:

$$\Box\rho + (e^2+m^2)\rho = 0 \tag{3.44}$$

as before, except for the fact that ρ is now a two–dimensional vector.

4 Supersymmetric generalization in $D = 4$

The Lagrangian (3.21) can be easily generalized to $D = 4$ $N = 1$ superspace by introducing a vector multiplet and a linear multiplet potential [9, 10]:

$$V = \overline{V}\,; \qquad L_\alpha \quad ; (\overline{D}_{\dot\alpha} L_\alpha) = 0 \tag{4.45}$$

through which the gauge invariant field strengths can be constructed [11] namely:

$$\begin{aligned} W_\alpha &= \overline{D}^2 D_\alpha V; && (4.46) \\ L &= i\left(D^\alpha L_\alpha - \overline{D}_{\dot\alpha}\overline{L}^{\dot\alpha}\right) && (4.47) \end{aligned}$$

which are invariant under:

$$\delta V = \Sigma + \overline{\Sigma}, \qquad \overline{D}\Sigma = 0 \tag{4.48}$$

$$\delta L_\alpha = \overline{D}^2 D_\alpha U, \qquad U = \overline{U} \tag{4.49}$$

respectively. Note that W^α and L satisfy the Bianchi identities:

$$D^\alpha W_\alpha = \overline{D}_{\dot\alpha} \overline{W}^{\dot\alpha}, \qquad D^2 L = \overline{D}^2 L = 0 \tag{4.50}$$

as a consequence of the identity:

$$D^\alpha \overline{D}^2 D_\alpha = \overline{D}_{\dot\alpha} D^2 \overline{D}^{\dot\alpha} \tag{4.51}$$

We observe that the combination $L_\alpha + m^{-1} W_\alpha$ is invariant under:

$$\delta L_\alpha = \overline{D}^2 D_\alpha U, \qquad \delta V = -mU \tag{4.52}$$

which is the supersymmetric generalization of the bosonic gauge invariance (for $I, \Lambda = 1$) of equations (1.5). The physical degrees of freedom of L_α are a scalar ϕ, a 2–form B and a Weyl spinor ζ, while the vector multiplet contains a vector A and a Weil spinor λ.
The supersymmetric generalization of (2.6 will be a $N = 1$ massive vector multiplet, containing a massive vector, a massive scalar and a massive Dirac spinor, all with mass $\mu = \sqrt{e^2 + m^2}$.
To derive this result we generalize to superspace the Lagrangian (3.21) by introducing two Lagrange multipliers $\psi_\alpha, (\overline{D}_{\dot\alpha} \psi_\alpha) = 0$ and $\Omega, (\Omega = {}^*\Omega)$ so that the action is [2]:

$$\begin{aligned} \mathcal{L} &= \int d^4\theta \left[-\frac{1}{2} L^2 + \Omega \left(L - i\mathcal{D}^\alpha \left(L_\alpha + m^{-1} W_\alpha \right) + i\overline{\mathcal{D}}_{\dot\alpha} \left(\overline{L}^{\dot\alpha} + m^{-1} \overline{W}^{\dot\alpha} \right) \right) \right] \\ &+ \left[\int d^2\theta \frac{1}{2} m(m + ie) \left(L^\alpha + m^{-1} W^\alpha \right) \left(L_\alpha + m^{-1} W_\alpha \right) + i\psi \left(W_\alpha - \overline{D}^2 D_\alpha V \right) \right. \\ &\quad \left. + \text{h.c.} \right]. \end{aligned} \tag{4.53}$$

If we vary with respect to ψ^α and Ω we get:

$$W_\alpha = \overline{D}^2 D_\alpha V \tag{4.54}$$

$$\begin{aligned} L &= i \left(\mathcal{D}^\alpha \left(L_\alpha + m^{-1} W_\alpha \right) - i\overline{\mathcal{D}}_{\dot\alpha} \left(\overline{L}^{\dot\alpha} + m^{-1} \overline{W}^{\dot\alpha} \right) \right) \\ &= i \left(\mathcal{D}^\alpha L_\alpha - \overline{\mathcal{D}}_{\dot\alpha} \overline{L}^{\dot\alpha} \right) \end{aligned} \tag{4.55}$$

[2]Note that in the case we have several fields L_I and W_α^Λ we easily find the the gauge invariance under (4.48) and (4.49) requires the condition (1.3).

which gives the Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\int d^4\theta L^2 + \frac{1}{2}\left(\int d^2\theta\ m(m+ie)L^\alpha L_\alpha + h.c.\right) \tag{4.56}$$

which is the supersymmetric generalization of the Lagrangian (2.6).

The dual supersymmetric Lagrangian is obtained instead by varying (4.53) with respect to L, L_α and W_α. One obtains:

$$L = \Omega, \qquad \psi = 0 \tag{4.57}$$

$$m(m+ie)(L_\alpha + m^{-1}W_\alpha) + i\overline{D}^2 D_\alpha\Omega = 0 \tag{4.58}$$

By insertion of (4.57) into (4.53) one obtains the dual Lagrangian:

$$\mathcal{L}_{Dual} = \frac{1}{2}\int d^4\theta\Omega^2 + \left(\frac{1}{2}\int d^2\theta \frac{m-ie}{m(m^2+e^2)}(\overline{D}^2 D^\alpha\Omega)^2 + h.c.\right) \tag{4.59}$$

The last (chiral) term gives:

$$\mathcal{L}_{Chiral} = \frac{1}{2}\int d^2\theta \frac{1}{(m^2+e^2)}\left[(W^\alpha W_a + h.c.) - i\frac{e}{m}(W^\alpha W_a - h.c.)\right] \tag{4.60}$$

The first term in (4.60) is the kinetic term of a massive vector superfield $(e^2+m^2)^{-\frac{1}{2}}W_\alpha$ while the last term is a total derivative which corresponds to the supersymetric generalization of the topological term $\frac{e}{2m}F\wedge F$, with θ –parameter $\frac{e}{m}$.

5 Conclusions

We have shown that $D/2$–forms in D even dimensions with both electric and magnetic mass terms are dual to massive $D/2-1$–forms with dyonic mass $\mu = \sqrt{e^2+m^2}$. This phenomenon has a supersymmetric generalization in $D=4$ for $N=1$ and $N=2$. In the latter case a tensor multiplet provides the dual version of the Higgs mechanism in which a hypermultiplet is eaten by a vector multiplet to combine into a long massive multiplet with mass μ. Under suitable assumptions on the e^I_Λ and $m^{I\Lambda}$ matrices, in the multivariable case the squared mass matrix becomes $\mu^2 = ee^T + mm^T$.

We observe that, when extended to interactions, the $N=2$ lagrangian with both e and m present are more general than the ones with either e or m

vanishing. In particular they may give rise to spontaneous supersymmetry breaking when suitably truncated to $N = 1$, as in the case of Calabi–Yau compactifications of Type IIB on orientifolds [12, 13, 14, 15]. In this case the GVW superpotential W [16], with e and m different from zero may lead to non trivial vacua solutions with vanishing vacuum energy and $N = 1 \rightarrow N = 0$ supersymmetry breaking. The resulting potential corresponds to an electric and magnetic Fayet–Iliopoulos term [17] whose $N = 2$ supergravity generalization was introduced in reference [5] for $I = 1$ and in [6] in the general case.

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